Using Response Surface Methodology to Build a Meta-Model for a Non-Linear Mixed Integer Lean Supply Chain Problem

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ABSTRACT

This paper describes how response surface methodology was used to build a meta-model based on the non-linear mixed integer model of Cruz and Kabiling (2005) that deals with a supply chain employing mechanisms of lean logistics. In this study, the experiments made use of the Central Composite Design. Three independent variables were considered, namely demand variability, holding cost, and transportation cost, as these came out as the most significant in the Plackett-Burman screening design. The following responses were recorded: total system cost, presence or absence of milk runs, number of open facilities, total system inventory, and number of ConWIP and Kanban routes. Regression models and response surfaces were developed and analyzed for each of the above. Finally, they were used to describe the supply chain environment, as characterized by the variations in costs, and demand behavior, where the use of particular lean principles are truly applicable from a cost reduction perspective, at least for the range of parameter values used in the designed experiments.

KEYWORDS: *Response surface methodology, meta-modeling, supply chain management, lean logistics*

I. INTRODUCTION

This paper describes how response surface methodology was used to build a meta-model based on results obtained from an operations research (OR) model. In particular, this study analyzes the non-linear mixed integer formulation of a supply chain employing mechanisms of lean logistics as presented in Cruz and Kabiling (2005). There are two reasons that one might want to undergo such an approach.

First, the structure of OR models and the size of expanded models when applied to existing supply chains present considerable computational difficulties. Some of the issues that arise from the solution process include scaling or the relative sizes of the initial and final variable values, initial solution or the starting values of the variables, and variable bounds or the allowable range that variables will be allowed to take as the solution progresses. This is especially the case when dealing with a non-linear mixed integer model. Adjusting the model to ensure a smooth run of solver engines can become an arduous task, as analyzing the results of an OR model is typically an iterative process requiring several runs. By building a meta-model, analysts can get an idea of the optimal solution in different supply chain environments, characterized by varying costs, demand, etc., without having to actually run the OR model.

Second, response surface methodology provides a tool for performing sensitivity analysis where traditional methods are no longer applicable. In particular, traditional approaches allow for changing model parameters one at a time while response surface methodology would allow for simultaneously varying as many parameters as desired. The idea is that if the cost coefficients in the objective function are treated as factors in the experiment, and the optimal objective function value is taken to be the response, then a response surface will be able to represent the appropriate value of the objective function, or any system variable that one would like to monitor, while changing model parameters such as costs, demand size and demand variability. This gives a more realistic and even strategic perspective when doing sensitivity analysis, as the aptness of the optimal solution may be scrutinized more holistically.

Furthermore, once the model is developed, it may be used to determine in what instances the supply chain environment, as characterized by the variations in costs, and demand behavior, makes the use of particular lean principles truly applicable from a cost reduction perspective.

The following are sub-problems that this study wishes to answer through analysis of the results obtained from the model:

- What cost and demand configurations make the use of milk runs desirable?
- How would one characterize the supply chain environment that merits the need to minimize the number of open facilities?
- In what instances do reducing total system inventory levels result in cost minimization?
- How do the cost and demand configurations affect the choice of either the Kanban or ConWIP pull system?
- What transportation cost, holding cost and demand configurations result in the highest overall desirability of a lean system characterized by minimal open facilities, minimal inventory levels, choice of pull system and the use of milk runs?

II. LITERATURE REVIEW

Giddings, Bailey, and Moore (2001) applied a similar methodology to a supply chain model developed as a mixed integer linear problem. Their study also employed response surface methodology, the purpose of which is to identify via design of experiments the functional relationships and the factor settings to optimize a certain response, that is the overall supply chain cost in their case. It begins with first-order designs to identify the most significant factors that affect supply chain costs, and a second phase that uses 2^{nd}-order designs to fit quadratic polynomials to the data. Giddings, Bailey, and Moore (2001) provide an example using data from PFS Logistics Consulting, a subsidiary of PepsiCo.

III. METHODOLOGY

In particular, the sensitivity analysis here makes use of the Central Composite Design, represented graphically in Figure 1. In the latter, each factor is varied over five levels, given by -α, -1, 0 (the center point), +α, and +1. Three independent variables were considered, those that came out as the most significant in the Plackett-Burman screening design. These are demand variability (labeled Factor A, or coded variable X_{dem}), holding cost (labeled Factor B, or coded variable $\underline{X_{hold}}$) and transportation costs (labeled Factor C, or coded variable X_{trans}). The design made use of a total of 20 experiments, six of which were center runs (referring to medium settings for each factor), and the remaining 14 experiments composed of one replicate each of factorial points (+1 for high , -1 for low) and axial points (+α, - α), with α=1.68179, as shown in Figure 1.

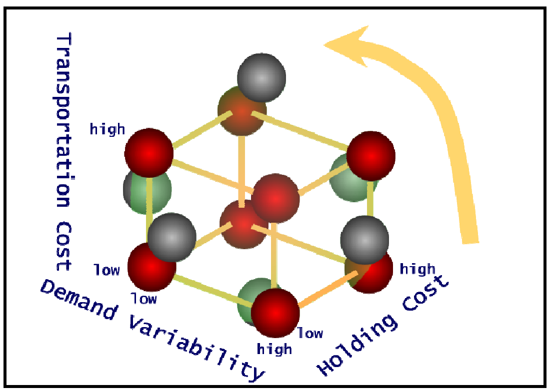

Figure 1: Central Composite Design

Table 1 provides some idea on the magnitude of these settings relative to other model parameters. Demand variability is measured relative to the demand rate, that is, the standard deviation is divided over the demand rate to yield the percentages in the table. In Hopp and Spearman (2000), this is called the coefficient of variation. Meanwhile, holding cost and transportation cost are related to backorder cost. The center point used for each factor is equivalent to the validation inputs used in the numerical example given by Cruz and Kabiling (2005). A complete listing of the actual factor levels used in the experiments are given in the Appendix.

The following responses were recorded in order to answer the sub-problems, and response surfaces were developed for each.

- Total system cost
- Presence or absence of milk runs
- Number of open facilities
- Total system inventory
- ConWIP and Kanban routes

Table 1: Characterization of Low, Medium and High Values for Changing Parameters

Parameter	**Low**	**Medium**	**High**
Demand Variability (relative to mean demand rate)	Std. Dev. 25% of the mean for product 1, 16.67% for product 2	Std. Dev. 50% of the mean for product 1, 33.33% for product 2	Std. Dev. 75% of the mean for product 1, 50% for product 2
RM Holding Cost per unit (relative to the medium backorder cost per unit)	250% for factory 1, 300% for factory 2, and 50% for factory 3	500% for factory 1, 600% for factory 2, 100% for factory 3	750% for factory 1, 900& for factory 2, 150% for factory 3
FG Holding Cost per unit (relative to the medium backorder cost per unit)	50% for factory 1 and 2, 75% for factory 3	100% for factory 1 and 2, and 150% for factory 3	150% for factory 1 and 2, 225% for factory 3
Base-stock Holding Cost per unit (relative to the medium backorder cost per unit)	75% for product 1 and 90% for product 2	150% for product 1 and 180% for product 2	225% for product 1 and 270% for product 2
Delivery Cost per unit (relative to the medium backorder cost per unit)	Average of 362.5% over all products, depots, cross-docks and customers	Average of 725% over all products, depots, cross-docks and customers	Average of 1087.5% over all products, depots, cross-docks and customers
Shipment Cost per unit (relative to the medium backorder cost per unit)	Average of 300% over all products, factories, depots and cross-docks	Average of 600% over all products, factories, depots and cross-docks	Average of 900% over all products, factories, depots and cross-docks

IV. RESULTS

IV.1 Total System Cost

Naturally, the lowest system costs are achieved when cost factors are at their lowest levels. Important to note in Figure 2 however is that the response surface for cost forms a ridge system, with contour lines almost parallel for high transportation costs. The effect of demand variability sets in when transportation costs are lowered.

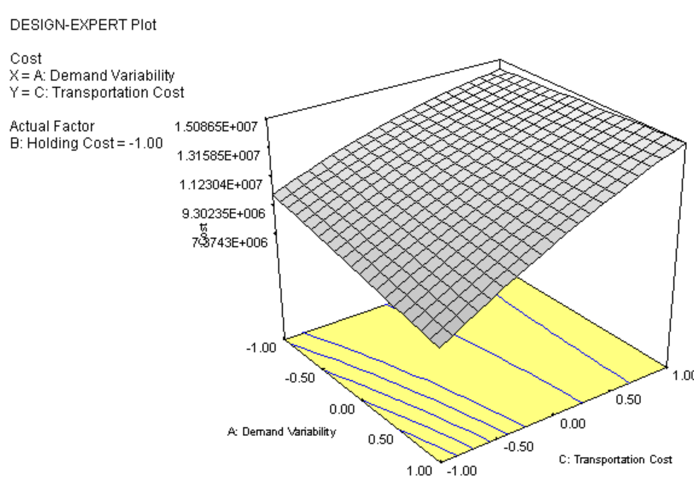

Figure 2: Response Surface for Total System Cost

The final model for cost, Y_{cost}, in hundred thousand monetary units, given in terms of coded factors is as follows:

$$Y_{cost} = 129 - 5.074X_{dem} + 5.796X_{hold} + 29.98X_{trans} - 5.819X_{trans}^2 + 8.584X_{dem}X_{trans} \quad \text{Eqn. 1}$$

IV.2 Milk Runs

None of the runs done for sensitivity analysis resulted in the use of milk runs. Thus for the range of parameter values used, the model deems milk runs generally undesirable. Neither a regression model nor a response surface can be developed in such a case. Attempts were made, through trial and error, to find a maximum milk run cost level where they would become desirable. However, lowering the milk run costs increased the number of instances that the non-linear sub-problems returned infeasible solutions, decreasing significantly the solver ability to find the optimal solution quickly.

Because of this, the approach for milk runs taken here is rather different from the approaches taken for all the other responses. Several binary variables were fixed such that the following conditions were forced to be part of the final solution given by the model:

- At least one factory, Factory 2, is open.
- The depot was forced open.
- Factory 2 supplies the depot with both types of products.
- The depot serves all four customers with both types of products.
- Factory 2 gets its supply from all suppliers via a milk run.
- The milk run for Factory 2 starts with Supplier 1 for Part 1, Supplier 2 for Part 2, and Supplier 3 for Part 3, before going back to the factory.

Given these forced conditions, it is certain that the final solution would involve at least one milk run. However, the model still retained some amount of flexibility as all continuous variables were left to vary during solver iterations. This means that the solver still had to determine appropriate stocking levels for raw materials, finished goods, depot base-stock, as well as milk run frequencies, actual service levels, etc. Furthermore, the model retained the option of opening other facilities and activating alternative routes in order to satisfy demand.

Afterwards, the model was run 20 times using the same set of changing parameter values used in all the other response surfaces developed in this chapter. The recorded response would be total cost, instead of number of milk runs. Design Expert, therefore, would evaluate desirability of a milk run based on its ability to reduce cost. The response surfaces for milk run desirability with low and high transportation costs are shown in Figures 3 and 4 respectively.

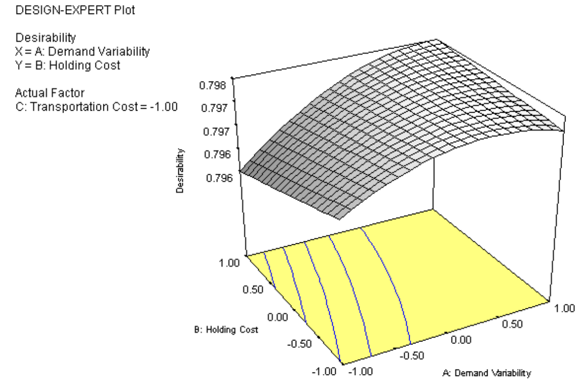

Figure 3:
Response Surface for Milk Run Desirability with Low Transportation Cost

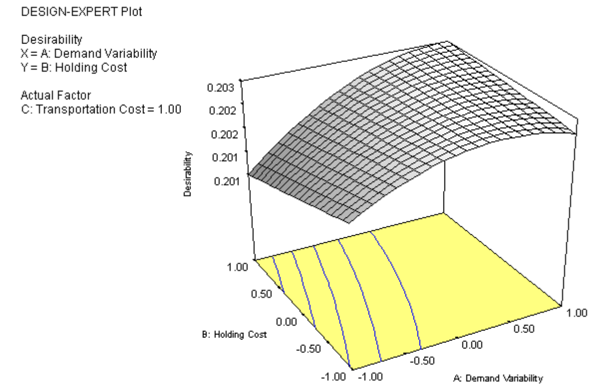

Figure 4:
Response Surface for Milk Run Desirability with High Transportation Cost

It can be observed that milk runs attain their highest desirability when demand variability is high and when holding costs are high. The advantage of milk runs is that they may be done more frequently than direct replenishment, as the latter would usually be subject to the capacity of the suppliers both to produce the units ordered and to make a number of replenishments in a given period of time. Frequent replenishment leads to lower inventory levels, which are very desirable in case holding costs are high.

Furthermore, although the shape of the response surface is maintained with varying transportation costs, the desirability of milk runs is greatly undermined when transportation costs are high. This may be explained by the nature of milk runs, where the company incurs the costs directly linked with bringing the units from the supplier to the factory. The system thus becomes susceptible to the brunt of high transportation costs.

The final regression for model with cost, in thousands of monetary units, as the response variable, having milk runs fixed, is as below:

$$Y_{\cos t} = 16470 - 9.524X_{dem} + 2.615X_{hold} + 4834X_{trans} + 7.109X_{dem}^{2} - 4.712X_{dem}X_{hold} \qquad \text{Eqn. 2}$$

IV.3 Number of Facilities

Figure 5 and 6 give the response surfaces based on number of facilities opened by the model for low and high demand variability respectively.

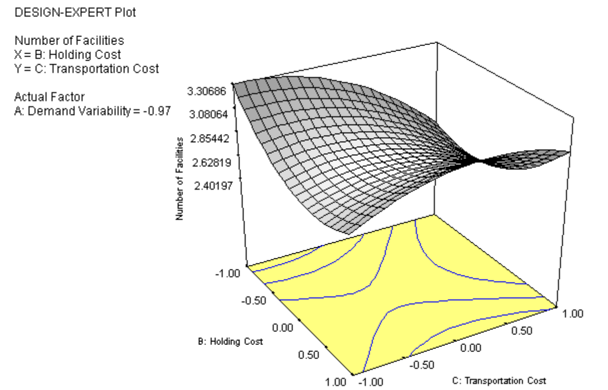

Figure 5:
Response Surface for Number of Facilities with Low Demand Variability

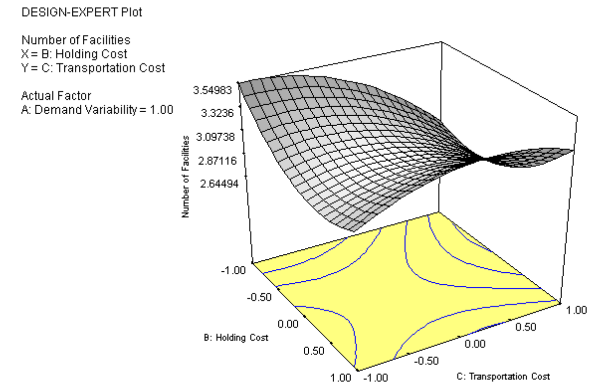

Figure 6:
Response Surface for Number of Facilities with High Demand Variability

A saddle point characterizes the response surface for the number of facilities in this case. The location of the saddle point can be easily spotted as one inspects the hyperbolic contour lines on the x-y plane. For medium holding cost, extreme values of transportation cost result in less facilities being opened. For medium transportation costs, the number of facilities increases when holding costs are varied to their extremes.

When demand variability is high, the response surface changes only slightly. The difference is that the area of the saddle point possesses less curvature than when demand variability is low, making the critical point less sensitive to changes in transportation and holding cost.

The final model for the number of open facilities, Y_{fac}, is given in coded factors as below:

$$Y_{fac} = 2.94 + 0.12X_{dem} - 0.15X_{trans} + 0.31X_{hold}^2 - 0.22X_{trans}^2 + 0.25X_{hold}X_{trans} \quad \text{Eqn. 3}$$

IV.4 Total System Inventory

When transportation costs are low, total system inventory is minimized when demand variability is low and holding costs are high. This is expected since low demand variability reduces the need for inventory capacity to buffer the demand fluctuations. In addition the high holding costs makes holding inventory undesirable anyway. The reverse is also true according to the response surface. Total inventory is maximized in an environment with high demand variability and low holding costs.

The response surface is similar in shape even when transportation costs are high, although it appears to be more flat. It can therefore be said that increasing transportation costs dulls the effect of changing demand variability and holding costs.

This may be explained by examining the benefits offered by low inventory. Holding fewer units in stock requires that more trips be made between facilities. So when transportation costs

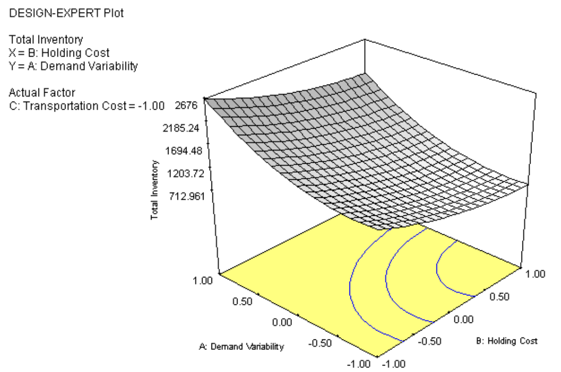

Figure 7:
Response Surface for Total System Inventory with Low Transportation Cost

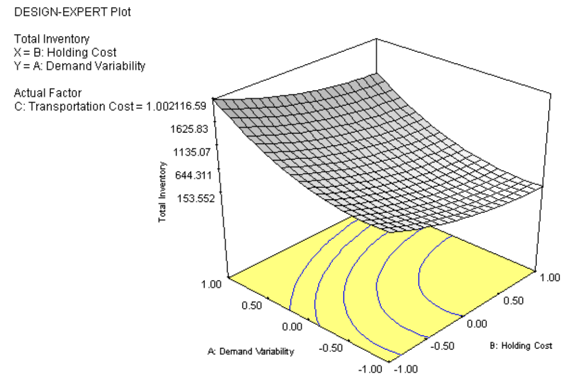

Figure 8:
Response Surface for Total System Inventory with High Transportation Cost

are low, then low inventory becomes highly favorable, that is, the cost savings in inventory exceeds the increased costs associated with frequent shipments.

However, when transportation costs are high, the model becomes rather indifferent, since lowering inventory with only be compensated by the significant cost increase for transportation. The final model for the response of total system inventory, Y_{inv}, given in coded factors is as follows:

$$Y_{inv} = 869.05 + 592.20X_{dem} - 380.79X_{hold} - 279.70X_{trans} + 376.39X_{dem}^2 + 177.87X_{hol}^2 \quad \text{Eqn. 4}$$

IV.5 ConWIP and Kanban Routes

Figures 9 and 10 present the response surfaces for the number of Kanban routes with high and low transportation costs respectively. It is observed that Kanban routes are activated more when demand variability is low and holding costs are high. ConWIP routes are selected more often when the situation is the reverse.

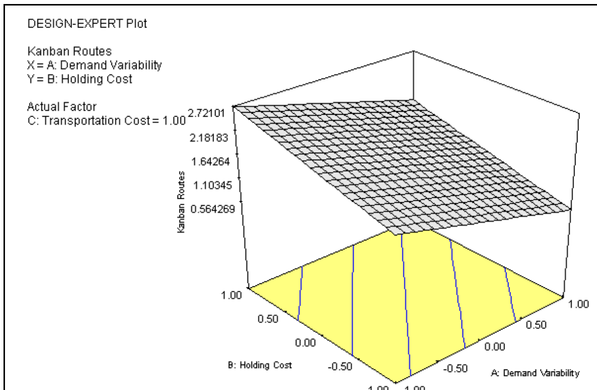

Figure 9:
Response Surface for Kanban Routes with High Transportation Cost

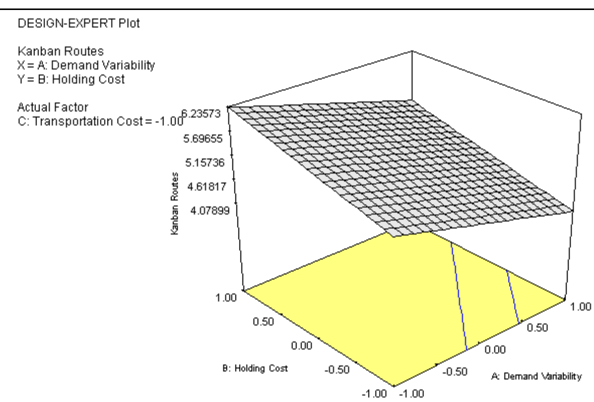

Figure 10:
Response Surface for Kanban Routes with Low Transportation Cost

IV.6 Overall Desirability of Lean Systems

The overall desirability of using a lean system as defined by the five parameters analyzed earlier can be done using the numerical optimization function of Design Expert based on the desirability response. The response surfaces for both low and high transportation costs are given in Figures 7.11 and 7.12.

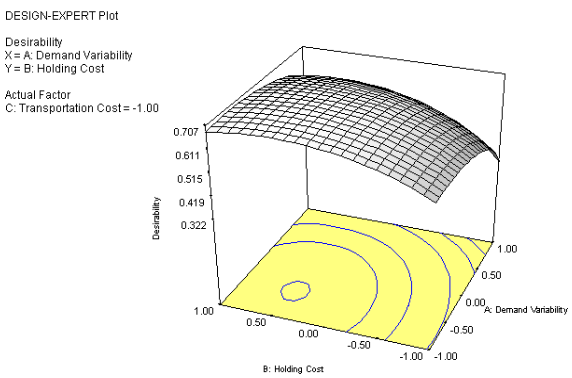

Figure 7.11:
Lean Desirability Response Surface in a Low Transportation Cost Environment

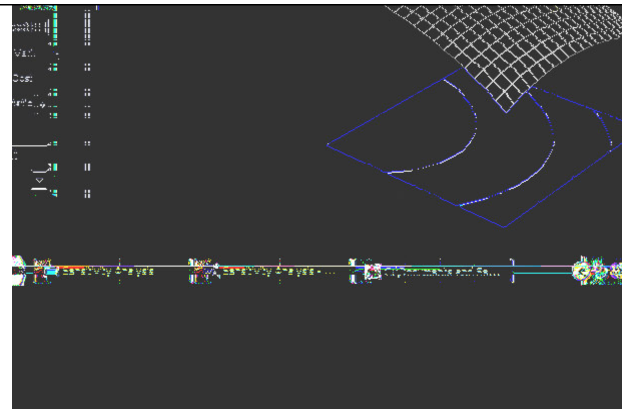

Figure 7.12:
Lean Desirability Response Surface in a High Transportation Cost Environment

When transportation cost is low, the response surface possesses a maximum, found in an area of high holding cost and low demand variability. This is expected, since a lean system promotes low inventory. Therefore, it is able to avoid the effects of high holding costs. On the other hand, holding little inventory makes the system more susceptible to fluctuations in demand, hence the higher desirability of lean systems in environments with low demand variability.

In a high transportation cost environment meanwhile, the maximum desirability shifts to an area of low demand variability and low holding costs as well. This is perhaps due to the tendency to reduce shipment frequency when transportation costs are high. In order to sustain service levels, therefore, higher levels of inventory are required, making the system more desirable in an area where holding costs are low.

V. CONCLUSION AND RECOMMENDATIONS

Through the approach described above, certain conclusions were made about the desirability of lean supply chains in relation to demand variability, transportation costs and holding costs, the factors that were statistically identified to have a significant effect on the optimal solution.

- For the range of parameter values used, the model deemed milk runs generally non-optimal. However, fixing the solution to include at least one milk run permits the evaluation of milk run desirability. Milk runs attain their highest desirability when demand variability is high and when holding costs are high. Furthermore, although the shape of the response surface is maintained with varying transportation costs, the desirability of milk runs is reduced significantly when transportation costs are increased.

- For constant holding cost, the number of open facilities finds a maximum at a certain level of transportation cost. Meanwhile, for constant transportation costs, the number of facilities is minimized at some level of holding cost. Increasing demand variability makes this saddle point less sensitive to changes in transportation and holding cost.

- Total inventory is minimized in an environment with low demand variability and high holding costs. In this regard, however, increasing transportation costs dulls the effect of changing demand variability and holding costs.

- Kanban routes are favored when demand variability is low and holding costs are high. ConWIP routes are favored in the reverse situation.

- The desirability of lean systems has much to do with the level of transportation costs. When transportation cost is low, lean systems are most desirable in an area of high holding cost and low demand variability. However, in a high transportation cost environment the maximum desirability shifts to an area of low demand variability and low holding costs.

By building a meta-model, analysts could get an idea of the optimal solution in different supply chain environments, characterized by varying costs, demand, etc., without having to actually run the OR model. RSM provided a tool for performing sensitivity analysis where traditional methods are no longer applicable. The approach could lead to a better understanding of the system behavior, lending to more enlightened decision-making and strategy formulation.

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Appendix: The following parameter values were used in this study. Notations in parentheses are as found in Cruz and Kabiling (2005). Low values are 50% less than the medium values, while high values are 150% higher.

Capital cost of opening factories ($CAPX^{fact}_{f}$)

	Low	Medium	High
Factory 1	750000	1500000	2250000
Factory 2	1600000	3200000	4800000
Factory 3	1750000	3500000	5250000

Fixed cost of setting up factory f for milk runs (C^{milk}_{f})

	Low	Medium	High
Factory 1	500	1000	1500
Factory 2	500	1000	1500
Factory 3	500	1000	1500

Capital cost of opening depots ($CAPX^{depot}_{d}$)

	Low	Medium	High
Depot 1	100000	200000	300000
Depot 2	1300000	2600000	3900000

Cost per unit collected for milk runs at factory f (C^{run}_{f})

	Low	Medium	High
Factory 1	5	10	15
Factory 2	5	10	15
Factory 3	5	10	15

Time between order and expected receipt of order at customer c for product p (DUE_{cp})

	Low		Medium		High	
	Product 1	*Product 2*	*Product 1*	*Product 2*	*Product 1*	*Product 2*
Customer 1	5	10	10	20	15	30
Customer 2	6	12	12	24	18	36
Customer 3	6.5	15	13	30	19.5	45
Customer 4	5	10	10	20	15	30

Desired service level for product p at customer c ($SERV_{cp}$)

	Low		Medium		High	
	Product 1	*Product 2*	*Product 1*	*Product 2*	*Product 1*	*Product 2*
Customer 1	0.35	0.4	0.7	0.8	1	1
Customer 2	0.35	0.4	0.7	0.8	1	1
Customer 3	0.35	0.4	0.7	0.8	1	1
Customer 4	0.35	0.4	0.7	0.8	1	1

Mean number of demand occurrences per unit time at customer c for product p (μ_{cp})

	Low		Medium		High	
	Product 1	*Product 2*	*Product 1*	*Product 2*	*Product 1*	*Product 2*
Customer 1	1	1.5	2	3	3	4.5
Customer 2	0.5	1	1	2	1.5	3
Customer 3	0.5	1	1	2	1.5	3
Customer 4	1	1.5	2	3	3	4.5

Standard deviation of the number of demand occurrences per unit time at customer c for product p (σ_{cp})

	Low		Medium		High	
	Product 1	*Product 2*	*Product 1*	*Product 2*	*Product 1*	*Product 2*
Customer 1	0.5	0.5	1	1	1.5	1.5
Customer 2	0.5	0.5	1	1	1.5	1.5
Customer 3	0.5	0.5	1	1	1.5	1.5
Customer 4	0.5	0.5	1	1	1.5	1.5

Annual holding cost per unit of part at factory f (CH^{RM})

	Low			Medium			High		
	Part 1	*Part 2*	*Part 3*	*Part 1*	*Part 2*	*Part 3*	*Part 1*	*Part 2*	*Part 3*
Factory 1	25	25	25	50	50	50	75	75	75
Factory 2	30	30	30	60	60	60	90	90	90
Factory 3	5	5	5	10	10	10	15	15	15

Annual holding cost per unit of product p at factory f (CH^{FG})

	Low		Medium		High	
	Product 1	*Product 2*	*Product 1*	*Product 2*	*Product 1*	*Product 2*
Factory 1	5	5	10	10	15	15
Factory 2	5	5	10	10	15	15
Factory 3	7.5	7.5	15	15	22.5	22.5

Annual holding cost per unit of work-in-process for product p at factory f (CH^{WIP})

	Low			Medium			High		
	Factory 1	*Factory 2*	*Factory 3*	*Factory 1*	*Factory 2*	*Factory 3*	*Factory 1*	*Factory 2*	*Factory 3*
Product 1	3.5	3.5	4	7	7	8	10.5	10.5	12
Product 2	3.5	3.5	4	7	7	8	10.5	10.5	12

Annual holding cost per unit of product p in depot d (CH^{STOCK})

	Low		Medium		High	
	Product 1	*Product 2*	*Product 1*	*Product 2*	*Product 1*	*Product 2*
Depot 1	6	10	12	20	18	30
Depot 2	7.5	9	15	18	22.5	27

Backorder cost per unit of product p per period for customer c (CB_{cp})

	Low		Medium		High	
	Product 1	*Product 2*	*Product 1*	*Product 2*	*Product 1*	*Product 2*
Customer 1	5	5	10	10	15	15
Customer 2	5	5	10	10	15	15
Customer 3	5	5	10	10	15	15
Customer 4	5	5	10	10	15	15

Fixed cost per shipment of product p from factory f to depot d (C^{ship}_{pfd})

		Low		Medium		High	
		Depot 1	*Depot 2*	*Depot 1*	*Depot 2*	*Depot 1*	*Depot 2*
Product 1	*Factory 1*	25	35	50	70	75	105
Product 2	*Factory 2*	25	35	50	70	75	105
Product 3	*Factory 3*	25	35	50	70	75	105
Product 1	*Factory 1*	25	35	50	70	75	105
Product 2	*Factory 2*	25	35	50	70	75	105
Product 3	*Factory 3*	25	35	50	70	75	105

Fixed cost per delivery of product p from depot d to customer c (C^{del}_{pdc})

		Low			
		Customer 1	*Customer 2*	*Customer 3*	*Customer 4*
Product 1	*Depot 1*	25	35	40	45
Product 1	*Depot 2*	25	35	40	45
Product 2	*Depot 1*	25	35	40	45
Product 2	*Depot 2*	25	35	40	45
		Medium			
		Customer 1	*Customer 2*	*Customer 3*	*Customer 4*
Product 1	*Depot 1*	50	70	80	90
Product 1	*Depot 2*	50	70	80	90
Product 2	*Depot 1*	50	70	80	90
Product 2	*Depot 2*	50	70	80	90
		High			
		Customer 1	*Customer 2*	*Customer 3*	*Customer 4*
Product 1	*Depot 1*	75	105	120	135
Product 1	*Depot 2*	75	105	120	135
Product 2	*Depot 1*	75	105	120	135
Product 2	*Depot 2*	75	105	120	135

Cost of installing a line for product p at factory f (C^{line}_{fp})

	Low		Medium		High	
	Product 1	*Product 2*	*Product 1*	*Product 2*	*Product 1*	*Product 2*
Factory 1	5000	5000	10000	10000	15000	15000
Factory 2	5000	5000	10000	10000	15000	15000
Factory 3	5000	5000	10000	10000	15000	15000