

# An Inventory Allocation Model Using Price Protection and Product Return Subsidies for Supply Chains under Declining Price Environments

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## ABSTRACT

Price protection is a credit given to distributors for unsold inventory when product prices decline during the planning horizon. Product return subsidies are given to distributors when products are returned by the distributors to the manufacturer. Both these subsidies are often implemented in the personal computer industry where the different members of the supply chain are exposed to risks associated with the rapid decline of product prices. This study investigates the interplay that occurs between these subsidies and other supply chain decisions when they are considered under a multi-period and multi-echelon setting under a declining price environment.

**KEYWORDS:** *Price Protection, Product Return Subsidy, Independent Distributors, Declining Price Environment*

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## I. INTRODUCTION

Supply chain research has gained a lot of attention in recent years. While the idea of the supply chain has been around from as early as the 1950's, advances in computing technology have now made supply chain research a lot more attractive and viable.

The personal computer (PC) industry makes for an interesting subject for supply chain research. The rapid rate at which new technology is introduced has produced an industry characterized by quick product obsolescence, significant price declines over the product cycle, and high demand uncertainty. (Lee, Padmanabhan, Taylor, & Whang, 2000) It is probably not surprising that the PC industry has come to make use of mechanisms for coordinating the

supply chain not commonly seen in other industries.

Price protection is a credit given to distributors for unsold inventory when product prices decline during the planning horizon. Some companies limit price protection either by putting a restriction on inventory eligible for credit based on when they were purchased or by limiting the magnitude of the price protection. (Lee, et al., 2000) Product return subsidies meanwhile are offered to distributors who return inventory to the manufacturer.

Even though it has become common practice to offer these subsidies in the PC industry, relatively little effort in the way of research has been made to investigate their actual impacts.

Lee et al. (2000) investigate how price protection affected the supply

chain. They approached the problem through the use of a two-step approach. The first step made use of a dynamic programming (DP) model that had an objective of maximizing the manufacturer's profit. The second step made use of a DP model that aimed to maximize the customer's profit when price protection was applied. Coordination was said to have been achieved when the manufacturer's profit was the same for both the first and second step. The study came to the following conclusion: price protection only achieved coordination when lead times were long. Otherwise, it was not enough to achieve coordination.

Taylor (2001) follows up with a similar study that added consideration for product return subsidies. Using the same two-step methodology as Lee et al. (2000), Taylor found that the use of both price protection and product return subsidies guaranteed supply chain coordination and a win-win situation for the manufacturer and distributor.

The two studies mentioned above share a few limitations however as they only considered two echelons and two periods. This means that the

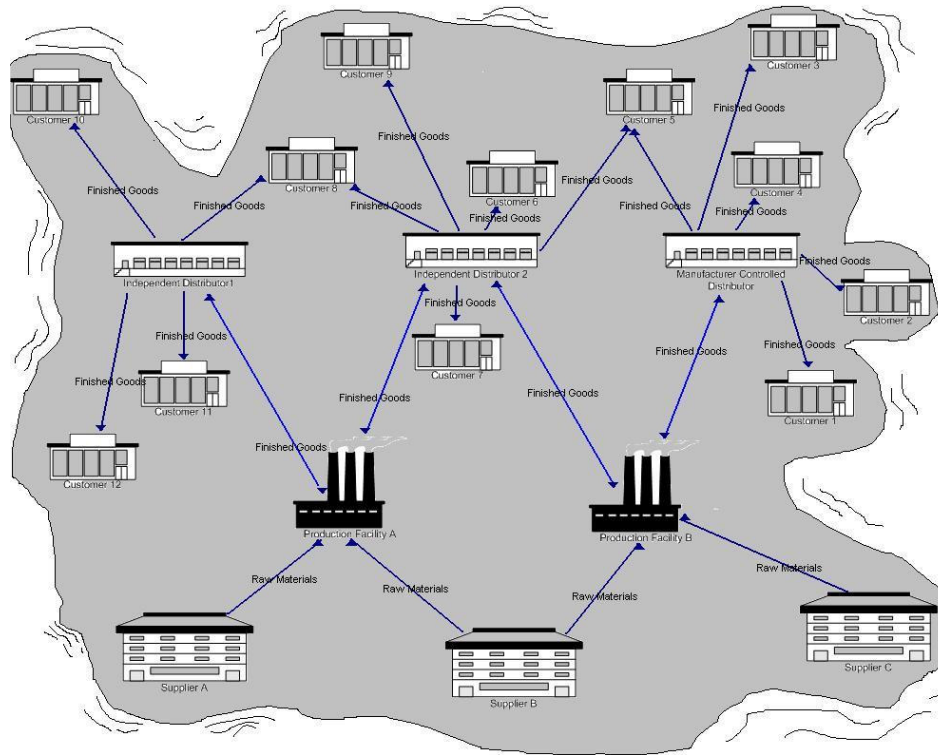
models that these studies produced could not be used to help make decisions involving participants elsewhere in the supply chain. Additionally, a two period planning horizon is rarely representative of how business cycles go.

One of the allures of modeling the supply chain lies in the sheer number of and interactions between the decisions to be made. For the purpose of tackling these aspects of the supply chain, the limitations of the two studies previously mentioned above make them unsuitable as the only basis of this study. More comprehensive research such as those by Kabiling (2005) and Azarias et al. (2006) make for a better fit in this capacity.

Both Kabiling (2005) and Azarias et al. (2006) consider multiple echelons and periods. However their studies implicitly assumed that the manufacturer controlled all distributors.

This study aims to come up with a multi-period and multi-echelon supply chain model that considers independent distributors. Figure 1 represents a rough sketch of how the supply chain would look like.

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**Figure 1:** Supply Chain Topology

The supply chain under consideration shall have four echelons with an echelon each allocated to the suppliers, production facilities, distributors and customers.

The suppliers are the source of the raw materials, which are transported from the suppliers to the production facilities for processing. From the production facilities, finished goods are then transported to the distributors, who are either independent or manufacturer controlled. They put finished goods in stock until such time they are needed by customers.

Independent distributors are differentiated from manufacturer-controlled distributors in order to identify which distributors qualify for price protection and product return subsidies.

Each entity in the supply chain is assigned a set of other entities in the preceding and/or succeeding echelon from which it is allowed to receive and/or send product deliveries.

## II. OBJECTIVES OF THE STUDY

The study aims to incorporate price protection and product return subsidies into a comprehensive multi-echelon and multi-period model under a declining price environment.

## III. METHODOLOGY

A non-linear programming model was formulated in order to achieve the goals of the study. The General Algebraic Modeling System (GAMS) was used to solve the model once it was set up. Model structure and response were tested to determine the resulting model's validity. Sensitivity analysis was then done to gain a better understanding of the model's behavior.

## IV. MODEL FORMULATION

### IV.1 Objective Function

The resulting model's objective function is one that maximizes the manufacturer's profit. The objective function is formulated as follows:

$$Max z = (revenue_j - cost_j)$$

Eqn.4.1.1

For each period  $j$ , revenue $_j$  is composed of revenue coming from independent distributors and customers. Costs $_j$  on the other hand is composed of the following variables: transportation costs, inventory costs, return handling costs, manufacturing costs, raw material costs, backorder costs and the cost of price protection and product return subsidies. As focus is given on two concepts: the costs of price protection and product return subsidies, equation 4.1.2 provides the computation of the cost of price protection; where  $csp_{ij}$  is the selling price to customers,  $FGINVD_{ijk}$  is the number of units of finished goods inventory and  $PPS_{ij}$  is the extent of price protection coverage.

Price protection subsidies encourage distributors to take in more inventories. This cost is incurred when the price of finished goods, as sold to independent distributors, drop (Lee et al., 2000).

$$\sum_i \sum_j \sum_{k \in dist(b)} (csp_{ij} - csp_{i(j-1)}) FGINVD_{ijk} PPS_{ij}$$

Eqn. 4.1.2

Product return subsidy costs, on the other hand, are associated with rebates or refunds given by the manufacturer to independent distributors for returning unsold products during a product's lifetime. Two types of product return subsidies are considered: a general one and a targeted one (Taylor, 2001).

The general product return subsidy means that all independent distributors opting to avail themselves of this strategy

receive the same percentage of the current price (as sold to independent distributors) of the products returned.

While, on the other hand, the product return subsidy is computed as the summation across all products, periods, independent distributors and their production facility sets, of the product of the current selling price to independent distributors  $isp_{ij}$ , the extent of general product return subsidies  $PRS_{ij}$ , and the number of units of products returned  $RGS_{ijkm}$ , as given in equation 4.1.3.

$$\sum_i \sum_j \sum_{k \in dist(b)} \sum_{m \in prodfac(k)} isp_{ij} PRS_{ij} RGS_{ijkm}$$

Eqn. 4.1.3

A disadvantage of the general product return subsidy is that the model will assign the same product subsidy coverage for all independent distributors. This entails that the pricing of the general product return subsidy is reliant on the distributor with the greatest profit shortfall, giving other independent distributors higher subsidies than needed to meet their profit goals. Targeted product return subsidies provide additional product return coverage to selected independent distributors. It acts as an intermediate to circumvent the disadvantage of implementing only product return subsidies.

The targeted product return subsidy is computed as the summation across all products, periods, independent distributors and their production facility sets, of the product of the current selling price to independent distributors  $isp_{ij}$ , the number of units of products returned  $TPRS_{ijkm}$ , and the sum of the extent of general product return subsidies  $PRS_{ij}$  and targeted product return  $TPRU_{ijk}$ . Given as,

$$\sum_i \sum_j \sum_{k \in dist(b)} \sum_{m \in prodfac(k)} isp_{ij} (TPRS_{ijk} + PRS_{ij}) TPRU_{ijkm}$$

Eqn. 4.1.4

## IV.2 Constraints

Since some of the distributors are independent, they operate with the expectation of profit. This feature is modeled through a constraint that requires the model to attain a certain level of profit for the independent distributors. The profit satisfaction constraint is modeled as:

$$\text{revenue}_j^{\text{dist}} - \text{cost}_j^{\text{dist}} + \text{subsidies}_j^{\text{dist}} \geq \text{profit}_j^{\text{dist}}$$

Eqn. 4.2.1

For each period  $j$ , revenue is composed of revenue coming from the sale of goods to customers of the independent distributors. Cost is composed of the following variables: transportation costs, inventory costs, return handling costs, manufacturing costs, raw material costs, and backorder costs. Price protection and product return subsidies are then added to the total to come up with the profit.

Inventory carry over constraints are used to ensure that any inventory left over from any given period are added to the next period's inventory. These constraints basically follow the format:

$$\text{invty}_{j+1} \leq \text{invty}_j + \text{del}_j^{\text{in}} - \text{del}_j^{\text{out}}$$

Eqn. 4.2.2

Where,

- $\text{invty}_j$ : starting inventory during period  $j$
- $\text{del}_j^{\text{in}}$ : deliveries received from suppliers\* during period  $j$
- $\text{del}_j^{\text{out}}$ : deliveries made to customers\* during period  $j$

For the production facilities,  $\text{invty}_j$  could represent either finished goods or raw material inventory. For finished goods inventory,  $\text{del}_j^{\text{in}}$  represents the goods manufactured by the production facility and product returns made by the distributors while  $\text{del}_j^{\text{out}}$  represents

the amount of finished goods delivered to the distributors.

For the raw material inventory,  $\text{del}_j^{\text{in}}$  represents the amount of raw materials that suppliers deliver to the facility; while  $\text{del}_j^{\text{out}}$  represents the amount of raw materials used to manufacture finished goods.

For the distributors,  $\text{invty}_j$  represents finished goods inventory.  $\text{del}_j^{\text{in}}$  represents the finished goods made by the production facilities to the distributors.  $\text{del}_j^{\text{out}}$  represents the goods delivered to the customers and those that were returned to the production facilities.

As can be seen in the report of Py (2008), other constraints considered were the demand satisfaction constraints, which ensured that the model assigned the proper amount of backorders and product deliveries; delivery constraints which limited the amount of goods that can be delivered by any entity in the supply chain to the amount it has on hand; and material component constraints which limited the amount of finished goods that can be produced by a production facility based on the amount of raw material on hand.

The manufacturing capacity constraint limited the amount of finished goods that a production facility can produce during any given period. Inventory capacity constraints limited the amount of inventory that the production facilities and distributors can carry at any given time.

The constraints for price protection and product return subsidies limited the value that these subsidies could assume from zero to one.

$$0 \leq \text{PRS}_{ij} \leq 1 \quad \forall i, j \quad \text{Eqn. 4.2.3}$$

$$0 \leq \text{TPRS}_{ijk} \leq 1 \quad \forall i, j, k \quad \text{Eqn. 4.2.4}$$

$$0 \leq \text{TPRS}_{ijk} + \text{PRS}_{ijk} \leq 1 \quad \forall i, j, k$$

Eqn. 4.2.5

\* The terms suppliers and customers are used to mean the source and destination of goods (e.g. for raw materials inventory of production facilities, suppliers here would mean the suppliers while customers would mean the amount of raw materials used to produce finished goods).

$$0 \leq PPS_{i,j} \leq 1 \quad \forall i,j \quad \text{Eqn. 4.2.6}$$

Finally, all variables were considered non negative.

## V. MODEL VALIDATION

Four scenarios were used in order to validate the model. The first scenario was the basic model where neither price protection nor product return subsidies were used. The second scenario was basically the first scenario with the implementation of price protection. The third scenario would have product returns implemented on top of the first scenario. The fourth scenario incorporated both price protection and product return subsidies into the model.

### V.1 Evaluation of Model Structure

Evaluation of model logic was done to examine the variables which were believed to have a significant effect on the system. Scenarios one to four were run under GAMS. The first scenario was then used as the reference point from which the succeeding scenarios were compared against to test model structure.

When implemented alone, price protection helped increase manufacturer profit by allowing the model to allocate more goods to the manufacturer. Interestingly, under price protection, independent distributors actually carried fewer inventories. This is because with subsidies coming in from manufacturers, independent distributors need not sell as much goods. This thus allows the model to allot more products to manufacturer controlled distributors while at the same time satisfying the profit goals of independent distributors.

On its own product return subsidies also helped increase manufacturer profit. It did this by allowing the manufacturer to deliver more inventories to independent distributors prior to demand peaks.

Implementing both price protection and product return subsidies had a similar effect of increasing manufacturer profit.

### V.2 Evaluation of Model Response

Testing the validity of model response usually involve comparing model response with actual system response. For hypothetical systems such as this one, the model's behavior can be compared to the behavior of other models from related or similar studies. For this study, Taylor's (2000) study was used for comparison.

Taylor's (2000) study claims that under certain conditions, price protection and product return subsidies allow for a win-win situation where both the distributors and the manufacturer earn more.

From the earlier evaluation of model structure, it is clear that manufacturers do earn more when price protection and product return subsidies are applied. For this section, the only remaining concern is whether or not these subsidies would also allow for distributors the chance to earn more. To this end, scenarios one to four were modified by turning the objective function to one that maximized independent distributor profit. Manufacturer profit was then turned into a constraint that needed to achieve the profit level obtained from scenario one.

From the results of the runs involving the modified models, it can be said that model response was valid since distributor profit increased with the implementation of a policy consisting of both price protection and product return subsidies.

## VI. SENSITIVITY ANALYSIS

For the sensitivity analysis, factors were first screened using design of experiments (DOE). Once the significant factors were determined, response surface methodology (RSM) was done to analyze model behavior when these factors were changed simultaneously. Three responses were identified for screening – manufacturer

profit, price protection subsidies and product return subsidies.

For these responses, the following factors were identified for screening: demand, finished goods inventory capacity, independent distributor profit, holding costs and transportation costs. 2-level factorial design was used and each of the factors was assigned low, medium and high values. Medium values were equivalent to the original values used. Low and high values were equal to 50% and 150% of the medium values respectively.

Scenario four was run 32 times with a different set of low, medium and high values to determine the significant factors. Table 1 summarizes the results of the DOE.

**Table 1:** Summary of Significant Factors per Response

<b>Response</b>	<b>Significant Factors</b>
<b>Manufacturer Profit</b>	<i>finished goods inventory capacity, distributor target profit, transportation costs</i>
<b>Price Protection Subsidies</b>	<i>finished goods inventory capacity, distributor target profit, holding costs</i>
<b>Product Return Subsidies</b>	<i>target profit</i>

With the significant factors identified, RSM was done using the central composite design. RSM was no longer done on product return subsidies since it only had one significant factor.

For manufacturer profit, the highest level was achieved when target independent distributor profit was at its lowest and demand was set to medium. This is logical since the manufacturer and the independent distributors are competing for sales to some customers. A lower independent distributor target profit would

therefore allow the model to allocate more goods to manufacturer controlled distributors and consequently achieve a higher profit for the manufacturer. As for manufacturer profit response to demand, this can be interpreted as a result of insufficient production capacity. Any system would have a cap on how much it can produce at any given time. When demand exceeds this limit, any unfulfilled demand becomes backorders that only translate to increased costs.

The desirability of product return subsidies increases as the finished goods inventory capacity increases. This is logical because product return subsidies act to encourage the return of goods to the manufacturer, however more inventory capacity encourage distributors to hold more inventory. PRS thus needs to be increased to counteract this. Holding cost on the other hand is not so linear in its effect. When inventory capacity is high, higher holding costs lead to lower PRS while lower holding costs lead to higher PRS. Again, this result is logical because higher inventory capacity discourages inventory retention while higher capacity encourages it.

## VII. RESULTS & CONCLUSIONS

Although this study allowed the extent of price protection to vary from zero to one, the model gave it a value of either zero or one. When there was a shortfall in independent distributor profit, the model simply increased the amount of inventory that the independent distributor held and gave it price protection coverage equal to the whole price drop. When there were no shortfalls, the model assigned a value of zero. This result is consistent with the objective of the model. Since the model maximized the profit of the manufacturer, it gave as much finished goods as possible to the manufacturer controlled distributor. In cases where there were shortfalls in independent distributor profit, any price protection coverage less than one would

necessitate the allocation of more inventories to independent distributors; thus decreasing manufacturer profit.

### **VII.1 Behavior of Manufacturer and Independent Distributor Profit**

Factors affecting manufacturer's profit are product return subsidies (PRS), demand, finished goods inventory capacity, independent distributor profit, and transportation costs. When distributor target profits are low, PRS when set to low improves manufacturer profit. PRS is most beneficial to manufacturer profit when it follows distributor profit.

Increases in independent distributor target profit, in turn, decrease manufacturer profit and increase product returns, PPS and PRS coverage. This is because the manufacturer needs to allocate more goods to independent distributors when distributor target profits are raised. Increase in finished goods inventory capacity allows increase of manufacturer profit when demand is less than average. The highest level of manufacturer profit is achieved when demand is set to medium. This means that with the way the system parameters has been set up, it could not handle demand much higher than normal.

### **VII.2 Behavior and Effect of Price Protection and Product Return Subsidies**

PPS is affected by the finished goods inventory capacity, distributor target profit, and holding costs; while PRS is affected by distributor target profit. Both PPS and PRS increase when distributor target profit increases. This suggests that PPS and PRS serve to protect manufacturer profit when distributor target profit increases.

Unlike PPS, PRS assumed a greater variety of values. Product return subsidies acted to increase independent distributor inventory. This is because the model made use of the independent distributors as a sort of storage facility in anticipation of demand peaks.

### **VII.3 Factors Affecting Product Returns**

The amount of products returned increases when either the target profit of independent distributors or customer demand increases. With rising demand, the manufacturer can use distributors for storage in preparation for demand peaks, as earlier mentioned. In such cases, PRS is used as a mechanism to encourage independent distributors to hold inventory for later retrieval by the manufacturer. Increased target profit increase product returns because PRS is dependent on it. With an increase in target profit, a greater amount of subsidy is needed to cover shortfalls.

## **VIII. RECOMMENDATIONS FOR FUTURE STUDIES**

Some of the routes those interested in extending this research may follow are listed below.

The study operates under the assumption of declining price environments. Future studies might want to investigate the effects of PPS and PRS when prices increase rather than decrease through time. Such a setting can be found in the pharmaceutical industry where the price of medicine increase as adoption increases. (Taylor, 2001)

Another interesting path is to approach the implementation of price protection and product return subsidies from the perspective of game theory. The study can be seen as a multi-objective problem that seeks to maximize both manufacturer and distributor profit. As can be seen from the results of the sensitivity analysis in chapter 6, these are two conflicting goals. Increasing one most often decreases the other. Those interested in taking such an approach might be interested in the work of Drake (2006), who investigated supply chain revenue sharing using game theory.



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